



ImplicitTerrain: a Continuous Surface Model for Terrain Data Analysis

Haoan Feng, Xin Xu, Leila De Floriani University of Maryland, College Park



Often used in geographical information systems (GISs) to model the Earth's **surface elevation and shape of the landscape**, to support various engineering, land-use planning, and environmental applications.

- Topographical analysis Local
- Topological analysis Global



Often used in geographical information systems (GISs) to model the Earth's surface elevation and shape of the landscape, to support various engineering, land-use planning, and environmental applications.

Topographical analysis based on local surface gradients & 2nd-order derivatives





Often used in geographical information systems (GISs) to model the Earth's surface elevation and shape of the landscape, to support various engineering, land-use planning, and environmental applications.

Topological analysis based on the overall shape of the surface, that can also be derived from **gradient**.





Basic Morse theory

Consider a scalar field *f* defined over a compact domain



Often used in geographical information systems (GISs) to model the Earth's surface elevation and shape of the landscape, to support various engineering, land-use planning, and environmental applications.

Key points & challenges:

- + Accurate surface elevation
- + Accurate surface derivatives
- + Support high resolution
- + Better 😊 "easy-to-understand"

Motivation: Discrete Surface Models are Limited



- Surface elevation and gradient quality is directly related to the **number of vertices** (grids or meshes).
- Surface gradient and high-order derivatives are **approximation** results.
- Analysis algorithms on these models are relatively **complex** with **special cases**.



Motivation: A Smooth Implicit Surface Model

- Terrain surface as a **2D scalar function**.
- Implicit Neural Representation (INR) based on *Coordinate-based Network* brings analysis back to the continuous world.
- C-n continuous surface model.
- Surface quality depends on **number of params**.
- "Over-smoothness" of INR is not a problem!





ImplicitTerrain: Pipeline Overview



ImplicitTerrain: Preprocessing



- **High-resolution**: 1000 x 1000 Raster data of digital elevation information of 1km² terrain
- Progressive fitting from *low-freq* to *high-freq* signals

ImplicitTerrain: Surface-plus-Geometry Model Fitting



- Progressive fitting from *low-freq* to *high-freq* signals
- Terrain surface analyses need pre-process the data (down-sample, smoothing, ...)
- Cascaded **surface model** (3, 256, sin) and **geometry model** (3, 256, sin)

ImplicitTerrain: Surface Model Analysis



Surface Model Analysis

- Derivatives calculated via **back propagation**
- Topological and topographical features can be derived just following their definitions
 - E.g., Critical point = function 1st-order derivative equals zero
 - E.g., Mean curvature:

$$H = \frac{(1+f_y^2)f_{xx} - 2f_x f_y f_{xy} + (1+f_x^2)f_{yy}}{2(1+f_x^2 + f_y^2)^{3/2}} \quad (5)$$

Name	Sizes (MBs)	Size ratio	Ψ_s PSNR	Ψ_s SSIM	Freq diff ×10	Grad norm diff ×10	Grad di- rection diff (rad) ×10	SPG PSNR	SPG SSIM
Swiss ₁	1.51/7.6	0.20	64.85	0.9999	1.49±2.31	0.54±0.52	0.62 ± 1.10	67.08	0.9999
Swiss ₂ Swiss ₃	1.51/7.6 1.51/7.6	0.20	60.53 59.75	0.9998	0.95 ± 2.08 0.13 ± 0.29	0.77 ± 1.00 0.86 ± 1.05	0.61 ± 0.77 0.72 ± 1.02	52.34 58.93	0.9992
Swiss ₃	1.51/7.6	0.20	62.54	0.9999	0.17 ± 0.32	0.56 ± 0.61	0.46 ± 0.57	66.59	0.9999

Table 2. Numerical evaluation of the fitting results of the real-world terrain. Sizes are the total model sizes and the input raster size, and Size ratio is their ratio. Ψ_s PSNR and Ψ_s SSIM are the fitting accuracy of the surface model to the smoothed data. SPG PSNR and SPG SSIM are the fitting accuracy of the SPG model to the original input. For the surface model, Freq diff is the mean and standard deviation of the frequency domain difference. Grad norm/direction diffs are the mean and standard deviation of the difference of gradient norm and direction between $\nabla \Psi_s$ and the estimated image gradient from I_s . ×10 denotes the scaling factor for better numerical representation.





Reconstructed data.





Name	Sizes (MRs)	Size	Ψ_s	Ψ_s	Freq diff	Grad	Grad di-	SPG	SPG
	(MDS)	ratio	FOINK	551IVI	×10	$\times 10$	(rad) $\times 10$	FOINT	55 11VI
Swiss ₁	1.51/7.6	0.20	64.85	0.9999	1.49 ± 2.31	0.54 ± 0.52	0.62 ± 1.10	67.08	0.9999
Swiss ₂	1.51/7.6	0.20	60.53	0.9998	0.95 ± 2.08	0.77 ± 1.00	0.61±0.77	52.34	0.9992
Swiss ₃	1.51/7.6	0.20	59.75	0.9998	0.13 ± 0.29	$0.86 {\pm} 1.05$	0.72 ± 1.02	58.93	0.9997
Swiss ₃	1.51/7.6	0.20	62.54	0.9999	$0.17 {\pm} 0.32$	0.56 ± 0.61	$0.46 {\pm} 0.57$	66.59	0.9999

Table 2. Numerical evaluation of the fitting results of the real-world terrain. Sizes are the total model sizes and the input raster size, and Size ratio is their ratio. Ψ_s PSNR and Ψ_s SSIM are the fitting accuracy of the surface model to the smoothed data. SPG PSNR and SPG SSIM are the fitting accuracy of the SPG model to the original input. For the surface model, Freq diff is the mean and standard deviation of the frequency domain difference. Grad norm/direction diffs are the mean and standard deviation of the difference of gradient norm and direction between $\nabla \Psi_s$ and the estimated image gradient from $I_{s.} \times 10$ denotes the scaling factor for better numerical representation.

• Accurate gradient field reconstruction



Name	Sizes (MBs)	Size ratio	Ψ_s PSNR	Ψ_s SSIM	Freq diff $\times 10$	Grad norm diff	Grad di- rection diff	SPG PSNR	SPG SSIM
		The Columb				$\times 10$	(rad) ×10		
Swiss ₁	1.51/7.6	0.20	64.85	0.9999	1.49 ± 2.31	0.54 ± 0.52	0.62 ± 1.10	67.08	0.9999
Swiss ₂	1.51/7.6	0.20	60.53	0.9998	0.95 ± 2.08	0.77 ± 1.00	0.61 ± 0.77	52.34	0.9992
Swiss ₃	1.51/7.6	0.20	59.75	0.9998	0.13±0.29	0.86 ± 1.05	0.72 ± 1.02	58.93	0.9997
Swiss ₃	1.51/7.6	0.20	62.54	0.9999	0.17±0.32	0.56 ± 0.61	0.46 ± 0.57	66.59	0.9999









(a) Separatrix lines - ImplicitTerrain.

(b) Separatrix lines - *Forman method*.

Name	precision	recall	$F_{0.5}$ score	WS_{ratio}
Synth _{ours}	1.00	1.00	1.00	0.68
Swiss ₁	0.90	0.96	0.91	0.17
Swiss ₂	0.91	0.831	0.89	0.31
Swiss ₃	0.89	0.78	0.87	0.69
Swiss ₄	0.91	0.83	0.89	0.35

Table 1. Topological analysis results of the synthetic and realworld terrain. WS_{ratio} between [0, 1] indicates the MIGs from both methods are well aligned.



(c) MIG - ImplicitTerrain.



(d) MIG - Forman method.

Figure 4. Comparison of topological analysis results of the synthetic terrain. Node colors and shapes represent the critical point types and the edge colors represent the separatrix lines as in the legend of (c) and (d). Better viewed in the digital version.

- On par noise robustness, also benefiting from the smoothing pre-processing
- Surface gradient field is more robust to noise than discrete method -> MIG structure preserved



Figure 8. Comparison of noise robustness. *Forman method* and ImplicitTerrain comparison via $F_{0.5}$ score and Wasserstein distance of the Swiss₁ w.r.t. noise level.

Ablation: SPG model vs. single model

- 4x faster fitting
- Much better accuracy (67 dBs vs. 58 dBs)
- Even faster (30x) if only surface model needed



Ablation: SPG model vs. single model

• Frequency domain comparison illustrates the higher efficiency of model parameter usage





ImplicitTerrain: a Continuous Surface Model for Terrain Data Analysis



For more details, please visit our project website: https://fengyee.github.io/implicit-terrain/



This work has been supported by the US National Science Foundation under grant number IIS-1910766.